Testing for Granger Non-Causality in Cointegrated Systems

Made Easy.

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Testing for Granger Non-Causality in Cointegrated Systems Made Easy.

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Abstract:

Considerable research has been devoted during the last five years to develop appropriate tests for Granger-Causality in integrated and cointegrated systems. Despite the existence of several tests, applied reasearch appears still to be conducted using some form of an F-test in the context of a VAR or an ECM. Reasons could be that the appropriate tests have only recently appeared in the literature, or that their implementation is relatively complex. This paper shows how to use readily available routines in RATS, SAS and SHAZAM to obtain the WALD test for Granger non-causality introduced by Toda and Yamamoto (1995).

JEL Classification: C12, C32.

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Testing for Granger Non-causality in Cointegrated Systems Made Easy.

1. Introduction

Testing for Granger non-causality in the context of stable VAR models involves testing whether some parameters of the model are jointly zero. In the past such testing has involved a standard F-test in a regression context.

However, recent research (see Toda and Phillips, 1993) has shown that when the variables are integrated, the F-test procedure is not valid, as the test statistic does not have a standard distribution.

Many years ago Pierce (1977) discussed the problem of ignoring the effect of autocorrelation. He concluded that “relationships will frequently tend to be ‘found’ that don’t exist ...” A similar problem occurs in this context. If inappropriate tests are used, the probability of Type I errors may well be (much) larger than the nominal size of the test. Hence, causal relationships which do not exist may be “discovered”.

Toda and Phillips’ result has given rise to the development of several alternative procedures. First, in cointegrated systems, an Error Correction model (ECM) can be transformed to its levels VAR form allowing a Wald type test (WALD) for linear restrictions to the resulting VAR model (WALD), see Lütkepohl and Reimers (1992) and Toda and Phillips (1993). Second, Mosconi and Giannini (1992) suggested a likelihood ratio test (LR) for systems that are cointegrated. Unfortunately, the virtues of simplicity and ease of application have been largely lost.

A third procedure (MWALD) (see Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996)) is theoretically very simple, as it involves estimation of a VAR model
augmented in a straightforward way (MWALD). A Monte Carlo experiment which included these three alternative test procedures, presented in Zapata and Rambaldi (1997), provides evidence that the MWALD test has comparable performance in size and power to the LR and WALD tests in samples of 50 or more observations. However, as presented in the literature, the implementation of the test is not entirely straightforward and involves some programming.

Despite the existence of these tests, applied research appears still to be conducted using some form of an F-test in the context of a VAR or an Error Correction Model (see for instance: Riezman, Whitman and Summers (1996), Hewarathna and Silvapulle (1996), Shan and Sun (1996), Erenburg and Wohar (1995), Mizala and Romaguera (1995), Saunders (1995), and Thorton (1995)). The failure to reach applied economists could be due to the fact that these tests have only recently appeared in the literature or to the relative complexity of implementation. Whatever the reason, the purpose of this paper is to show how the theoretical simplicity of MWALD can be matched by easy computation using the standard facilities of common computer packages, without the need for programming. Thus, the virtue of the practical simplicity of the F-test can be re-captured when the variables are integrated or cointegrated.

The presentation is as follows: Section two sets out the notation and presents the MWALD test. Section three proves that the quadratic form required to compute the MWALD test is numerically identical to the chi-squared test obtained by estimating the model as a set of Seemingly Unrelated Regressions. Section four shows how to use the SUR routines in SHAZAM, SAS and RATS to obtain the MWALD test. Section five provides a brief summary.
2. The MWALD test for non-causality

Toda and Yamamoto (1995) proved that in integrated and cointegrated systems the Wald test for linear restrictions on the parameters of a VAR(k) has an asymptotic $\chi^2$ distribution when a VAR($k + d_{\text{max}}$) is estimated, where $d_{\text{max}}$ is the maximum order of integration in the system.

In order to clarify the principle, let us consider the simple example of a bivariate ($p=2$) model, with one lag ($k=1$). That is,

$$x_t = A_o + A_1 x_{t-1} + e_t,$$

or more fully,

$$
\begin{bmatrix}
    x_{1t} \\
    x_{2t}
\end{bmatrix}
= 
\begin{bmatrix}
    a_{10} \\
    a_{20}
\end{bmatrix} + 
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
    x_{1,t-1} \\
    x_{2,t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
    e_{1t} \\
    e_{2t}
\end{bmatrix}
$$

where $E(e_t) = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = 0$ and $E(e_t e_t') = \Sigma$.

To test that $x_2$ does not Granger cause $x_1$, we will test the parameter restriction $a_{12}^{(1)} = 0$. If now we assume that $x_{1t}$ and $x_{2t}$ are I(1), a standard t-test is not valid. Following Dolado and Lütkepohl (1996), we test $a_{12}^{(1)} = 0$ by constructing the usual Wald test based on least squares estimates in the augmented model:

$$
\begin{bmatrix}
    x_{1t} \\
    x_{2t}
\end{bmatrix}
= 
\begin{bmatrix}
    a_{10} \\
    a_{20}
\end{bmatrix} + 
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
    x_{1,t-1} \\
    x_{2,t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
    a_{11}^{(2)} & a_{12}^{(2)} \\
    a_{21}^{(2)} & a_{22}^{(2)}
\end{bmatrix}
\begin{bmatrix}
    x_{1,t-2} \\
    x_{2,t-2}
\end{bmatrix}
+ 
\begin{bmatrix}
    e_{1t} \\
    e_{2t}
\end{bmatrix}
$$

We will now set down the MWALD test procedure for a general VAR($k$) model.

Let $x_t$ be a vector of $p$ economic variables, $x_{1t}, \ldots, x_{pt}$, satisfying a VAR($k$) process. Then,

$$x_t = A_o + A_1 x_{t-1} + \ldots + A_k x_{t-k} + e_t, \quad t = k+1, \ldots, T \quad (1)$$
where $e_{k+1}, ..., e_T$ are ~ iid $(0, \Sigma)$.

Defining $A = [A_0 \ A_1 \ ... \ A_k]$ equation (1) can be written in the form:

$$x_t = A x_{1_t} + e_t, \quad t = k+1, \ldots, T$$

(2)

where,

$$x_{1_t} = [1 \ x'_{t-1} \ \ldots \ x'_{t-k}]'$$

(3)

Finally, concatenating observations horizontally by defining

$$x = [x_{k+1} \ x_{k+2} \ \ldots \ x_T]$$

$$x_{1} = [x_{1_{k+1}} \ x_{1_{k+2}} \ \ldots \ x_{1_T}]$$

$$e = [e_{k+1} \ e_{k+2} \ \ldots \ e_T]$$

equation (2) can be written as

$$x = A x_1 + e$$

(4)

The estimate $\hat{A}$ and the variance-covariance, $\Sigma_A$, can be obtained using multivariate least squares. The estimator of $A_v$, where $A_v = \text{vec}(A)$, is (see Lütkepohl, 1991):

$$\hat{A}_v = ((x_1 x_1')^{-1} \otimes I_p) \text{vec}(x)$$

and the estimated variance-covariance of $\hat{A}_v$ is given by

$$\hat{\Sigma}_A = \Gamma^{-1} \otimes \hat{\Sigma},$$

(5)

where,

$$\Gamma = (x_1 x_1')/T^*$$

(6)

$$\hat{\Sigma} = x M x'/T^*$$

$$M = I_{T^*} - x_1'(x_1 x_1')^{-1} x_1$$

(7)

and,
$T^* = T - k.$

To compute the MWALD test we proceed as follows:

(a) Estimate a VAR($k+d_{\text{max}}$) process by multivariate least squares\(^2\), where $d_{\text{max}}$ is the maximum degree of integration in the system, to obtain $\hat{A}_v 0$, the least squares estimate of $A_v$.

(b) Estimate $\Sigma_A$ using (5).

(c) Let $R$ be a $J \times (pk + 1)$ matrix which selects the appropriate ‘non-causality’ parameters.

Then, under the null of non-causality

$$\lambda_w = T(R\hat{A}_v)' (R \hat{\Sigma}_A R')^{-1} (R\hat{A}_v) 0$$

has an asymptotic $\chi^2_{(J)}$ distribution.

In the bivariate example presented above, to test that $x_2$ does not Granger cause $x_1$, we must test:

$$H_0: a_{12}^{(i)} = 0$$

or

$$H_0: R A_v = 0,$$

where $R$ is given by

$$R = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]$$

and $A_v$ by

$$A_v = [a_{10} \ a_{20} \ a_{11}^{(1)} \ a_{21}^{(1)} \ a_{12}^{(1)} \ a_{22}^{(1)} \ a_{11}^{(2)} \ a_{21}^{(2)} \ a_{12}^{(2)} \ a_{22}^{(2)}]'$$

\(^2\) A likelihood ratio test to empirically determine the value of ‘$k$’ is detailed in Enders (1995) pages 312-315.
3. The MWALD test in the SUR framework

The test statistic $\lambda_{w}$, given in (8) has been obtained by organising the observations side-by-side, as presented in (4). An alternative organisation of the data is to stack the observations vertically. This is achieved by transposing (4) to obtain

$$x' = x_1' A' + e'$$

(9)

If we vectorise (9), we have

$$\text{vec}(x') = (I_p \otimes x_1') \text{vec}(A') + \text{vec}(e')$$

(10)

We now introduce the following notation:

$$\alpha = \text{vec}(A')$$

$$y = \text{vec}(x')$$

$$u = \text{vec}(e')$$

enabling (9) to be written as

$$y = (I_p \otimes x_1') \alpha + u$$

(11)

where,

$$E(uu') = \Sigma \otimes I_T$$

(12)

The vector $y$ consists of observations on $x_1$, followed by observations on $x_2$ and so on. The vector $\alpha$ is simply a reordering of the parameter vector $A_v$. Equations (11) and (12) are a system of Seemingly Unrelated Regressions (SUR) (See Judge et al., 1988).

By standard theory (see, for example, Lütkepohl, 1991) the variance-covariance matrix $\Sigma_\alpha$ of the least squares estimator $\hat{\alpha}$ is estimated by

$$\hat{\Sigma}_\alpha = \hat{\Sigma} \otimes [(x_1 x_1') / T']^{-1}$$

(13)

where $\hat{\Sigma}$ is defined in (6) and (7).
Suppose now that the matrix $S$ selects and reorders elements of $\hat{\alpha}$ so that

$$S \hat{\alpha} = R \hat{A}_{\gamma},$$

then, it follows that

$$S \hat{\Sigma}_{\alpha} S' = R \hat{\Sigma}_{\lambda} R',$$

and therefore

$$(S\hat{\alpha})'[S\hat{V}(\hat{\alpha})S']^{-1}(S\hat{\alpha}) = \lambda_w$$

where $\lambda_w$ is defined in (8).

It is seen from (11) that the regressor variables in each equation of the SUR system are identical (namely, $x_1'$) and so the least squares estimate $\hat{\alpha}$ is also the SUR estimate (see Judge et al., 1988). The form of $\hat{\Sigma}_{\alpha}$ given by (13) is also easily verified to be the estimated variance-covariance matrix of the SUR estimator. Thus, $\lambda_w$ is just the Wald statistic for testing the restriction $S\alpha = 0$ in the SUR system (11). This test can be routinely computed by several of the available commercial econometric packages (see Section 4.).

Thus, the test for Granger non-causality becomes computationally very simple. Each variable is regressed on every variable lagged from one to $k+d_{\text{max}}$ lags in a SUR system, and the restriction $S\alpha = 0$ is tested. It should be noted that if the computer package produces an F-statistic, then $\lambda_w = JF$. 
4. Using RATS, SAS, and SHAZAM to compute the MWALD

To demonstrate the computation of the MWALD test we use a system of three (p = 3) variables y, z and w. The data set contains 100 observations on three I(1) variables. Pre-testing indicates that k = 4.

Let us test that z does not Granger-cause y. Because \( d_{\text{max}} = 1 \), we must estimate a VAR(5) and test that \( z_{t-1}, z_{t-2}, z_{t-3} \) and \( z_{t-4} \) do not appear in the \( y_t \) equation. The system to be estimated is:

\[
\begin{bmatrix}
    y_t \\
    z_t \\
    w_t 
\end{bmatrix} =
A_0 + A_1 \begin{bmatrix}
    y_{t-1} \\
    z_{t-1} \\
    w_{t-1} 
\end{bmatrix} + A_2 \begin{bmatrix}
    y_{t-2} \\
    z_{t-2} \\
    w_{t-2} 
\end{bmatrix} + A_3 \begin{bmatrix}
    y_{t-3} \\
    z_{t-3} \\
    w_{t-3} 
\end{bmatrix} + A_4 \begin{bmatrix}
    y_{t-4} \\
    z_{t-4} \\
    w_{t-4} 
\end{bmatrix} + A_5 \begin{bmatrix}
    y_{t-5} \\
    z_{t-5} \\
    w_{t-5} 
\end{bmatrix} + \begin{bmatrix}
    e_y \\
    e_z \\
    e_w 
\end{bmatrix}
\]

and the null hypothesis is

\[ H_0: a_{12}^{(1)} = a_{12}^{(2)} = a_{12}^{(3)} = a_{12}^{(4)} = 0 \] (14)

where \( a_{12}^{(i)} \) are the coefficients of \( z_{t-i}, i = 1, \ldots, 4 \), in the first equation of the system.

If we were to compute the MWALD test as explained in Section 2., we would have to program the corresponding matrices. To illustrate, we have programmed the steps in GAUSS, a well known matrix oriented software, so that the results can be compared to the SUR output from RATS, SAS and SHAZAM. The GAUSS program and output is included in Appendix A.1 for the reader’s reference. The value of \( \lambda_w \) is in bold.

Below we present the codes for testing the hypothesis (14) using RATS, SAS and SHAZAM. Complete output is presented in Appendix A.2 with the computed values of \( \lambda_w \) (F in SAS case) in bold.
4.1 RATS

EQUATION YEQ Y
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

EQUATION ZEQ Z
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

EQUATION WEQ W
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

SUR 3
# YEQ
# ZEQ
# WEQ
TEST(PRINT)
# 7 8 9 10
# 0 0 0 0

4.2. SAS

Assuming the lagged values of y, z and w have been created previously (see Appendix A.2.2 for details) the code is:

PROC SYSLIN SUR VARDEF=N;
TITLE "WALD TEST USING THE SUR PROCEDURE";
Y:MODEL Y = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
  YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
Z:MODEL Z = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
  YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
W:MODEL W = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
  YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
STEST Y.ZLAG1=0, Y.ZLAG2=0, Y.ZLAG3=0, Y.ZLAG4=0;
RUN;

Note that in SAS the value of $\lambda_w = J \times \text{‘numerator’}$, that is the value labelled ‘numerator’ multiplied by the number of restrictions (J).

4.3 SHAZAM

Assuming the lagged values of y, z and w have been created previously (see Appendix A.2.3 for details) the code is:

sample 6 100
system 3 / dn
ols y ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols z ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols w ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
test
test lz:1=0
5. Summary

This paper has presented a method for the practical implementation of the Toda and Yamamoto (1995) Wald test for Ganger non-causality in integrated and cointegrated systems. We have shown that the numerical value of the required Wald test can be obtained by using the Seemingly Unrelated Regressions routine readily available in econometrics packages. The codes for RATS, SAS and SHAZAM have been included for the reader’s future reference.
6. References


Appendix

A.1 Computing MWALD using matrix manipulation

(1) Program

```plaintext
/* COMPUTING THE WALD TEST OF TODA AND YAMAMOTO(1995) AND
* DOLADO AND LUTKEPOHL (1996).
* THIS PROGRAM HAS BEEN WRITTEN IN GAUSS.
* LINES BETWEEN "**********" AND MARKED "<<<<<" NEED TO BE CHANGED
* ACCORDING TO DATA SET
*****************************************************************/
load
dat[100,3] = example.dat;
y = dat[.,1];
z = dat[.,2];
w = dat[.,3];
p = 3;
k = 5;
t = rows(dat) - k;

x = dat[k+1:rows(dat),1:p]'; /* VECTOR OF DEPENDENT VARIABLES */
ly = zeros(rows(dat),k);
lz = zeros(rows(dat),k);
lw = zeros(rows(dat),k);

i = 1;
do while i <= k;
  ly[.,i] = lagn(y,i);
  lz[.,i] = lagn(z,i);
  lw[.,i] = lagn(w,i);
  i = i + 1;
endo;

x1 = zeros(k*p+1,t);
x1[1,.] = ones(1,t);

i = 1; j = 4;
do while i <= k;
  x1[i+j:i*p+1,.] = (ly[k+1:rows(dat),i]') |
                  (lz[k+1:rows(dat),i]') |
                  (lw[k+1:rows(dat),i]');
  j = j + 2;
  i = i + 1;
endo;

output file = mwald.out reset;

"+++++++++++WALD TEST IN AUGMENTED VAR +++++++++++++++++++++++"

vahat = ((inv(x1*x1'))*x1) .* eye(p) .* vec(x);
```

g = (1/T)*(x1*x1');
i = eye(T);
o = (1/T)* x*(i-x1'*inv(x1*x1')*x1)*x';
siga = inv(g) .*. o;

r = zeros(4,48);            /* <<<<<<<<< CHANGE ACCORDINGLY */
r[1,7]  = 1;                /* <<<<<<<<< CHANGE ACCORDINGLY */
r[2,16] = 1;                /* <<<<<<<<< CHANGE ACCORDINGLY */
r[3,25] = 1;                /* <<<<<<<<< CHANGE ACCORDINGLY */
r[4,34] = 1;                /* <<<<<<<<< CHANGE ACCORDINGLY */

l = t*(r*vahat)'*inv(r*siga*r')*(r*vahat);

" Vector Ahat = " vahat;
"Value of MWALD = " l;
output off;
end;

(2) Output

++++++++++++WALD TEST IN AUGMENTED VAR +++++++++++++++++++++++
VAR(k+1) OUTPUT

Vector Ahat =
-0.135319
 0.089166
-0.094066
 0.610487
 0.037054
 0.054125
-0.108172
 1.151503
 0.297994
 0.094815
-0.262564
 0.253337
-0.119507
-0.332908
-0.335877
-0.436204
-0.297336
-0.424655
 0.403291
 0.508831
 0.648132
 0.134911
 0.484571
 0.667378
 0.090342
 0.029946
 0.081160
-0.064501
-0.325685
-0.461509
-0.073140
 0.150421
 0.091913
 0.122368
A.2 Computing MWALD using the SUR routine

A.2.1 RATS

(1) Program

CAL 1960 1 4
ALL 10 1990:4
OPEN DATA EXAMPLE.DAT
DATA(FORMAT=FREE, ORG=OBS) / Y Z W

EQUATION YEQ Y
# CONSTANT Y(1 2 3 4 5) Z(1 2 3 4 5) W(1 2 3 4 5)

EQUATION ZEQ Z
# CONSTANT Y(1 2 3 4 5) Z(1 2 3 4 5) W(1 2 3 4 5)

EQUATION WEQ W
# CONSTANT Y(1 2 3 4 5) Z(1 2 3 4 5) W(1 2 3 4 5)

SUR 3
# YEQ
# ZEQ
# WEQ
TEST(PRINT)
# 7 8 9 10
# 0 0 0 0

(2) Output

| Dependent Variable Y - Estimation by Seemingly Unrelated Regressions |
|-------------------------|---------------------|---------------------|
| Quarterly Data From 1961:02 To 1984:04 | Usable Observations: 95 | Degrees of Freedom: 79 |
| Mean of Dependent Variable: -4.393564211 | Std Error of Dependent Variable: 1.360319578 |
| Centered R**2: 0.961389 | R Bar **2: 0.954057 |
| Uncentered R**2: 0.996655 | T x R**2: 94.682 |
| Sum of Squared Residuals: 6.7162207997 |
| Durbin-Watson Statistic: 2.01857 |

Value of MWALD = 16.965899
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff</th>
<th>Std Error</th>
<th>T-Stat</th>
<th>Signif</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.133518861</td>
<td>0.099151193</td>
<td>-1.36477</td>
<td>0.17232444</td>
</tr>
<tr>
<td>Y(1)</td>
<td>0.610487113</td>
<td>0.163603508</td>
<td>3.73150</td>
<td>0.00019034</td>
</tr>
<tr>
<td>Y(2)</td>
<td>-0.119506609</td>
<td>0.190519196</td>
<td>-0.62727</td>
<td>0.53048353</td>
</tr>
<tr>
<td>Y(3)</td>
<td>0.134910599</td>
<td>0.190327421</td>
<td>0.70883</td>
<td>0.47842733</td>
</tr>
<tr>
<td>Y(4)</td>
<td>-0.073139742</td>
<td>0.191866565</td>
<td>-0.38120</td>
<td>0.70305405</td>
</tr>
<tr>
<td>Y(5)</td>
<td>-0.108223530</td>
<td>0.150044267</td>
<td>-0.72128</td>
<td>0.47073990</td>
</tr>
<tr>
<td>Z(1)</td>
<td>0.094814777</td>
<td>0.153354242</td>
<td>0.61827</td>
<td>0.53639544</td>
</tr>
<tr>
<td>Z(2)</td>
<td>0.403291237</td>
<td>0.182999217</td>
<td>2.20379</td>
<td>0.02753935</td>
</tr>
<tr>
<td>Z(3)</td>
<td>0.122367785</td>
<td>0.181941134</td>
<td>0.67257</td>
<td>0.50122216</td>
</tr>
<tr>
<td>Z(4)</td>
<td>-0.258382550</td>
<td>0.147597829</td>
<td>-1.75059</td>
<td>0.08001742</td>
</tr>
<tr>
<td>Z(5)</td>
<td>-0.006420483</td>
<td>0.149921769</td>
<td>-0.04283</td>
<td>0.96584059</td>
</tr>
</tbody>
</table>

Dependent Variable 2 - Estimation by Seemingly Unrelated Regressions
Quarterly Data From 1961:02 To 1984:04
Usable Observations 95 Degrees of Freedom 79
Centered R**2 0.963283 R Bar **2 0.956312
Uncentered R**2 0.986081 T x R**2 93.678
Mean of Dependent Variable -2.255310147
Std Error of Dependent Variable 1.7771535787
Sum of Squared Residuals 10.831622312
Durbin-Watson Statistic 2.109382

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff</th>
<th>Std Error</th>
<th>T-Stat</th>
<th>Signif</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.089165972</td>
<td>0.125916376</td>
<td>0.70814</td>
<td>0.47886054</td>
</tr>
<tr>
<td>Y(1)</td>
<td>0.037054231</td>
<td>0.207767150</td>
<td>0.17834</td>
<td>0.85845204</td>
</tr>
<tr>
<td>Y(2)</td>
<td>-0.332908069</td>
<td>0.241948543</td>
<td>-1.37595</td>
<td>0.16883843</td>
</tr>
<tr>
<td>Y(3)</td>
<td>0.484570553</td>
<td>0.241705000</td>
<td>2.00480</td>
<td>0.04498427</td>
</tr>
<tr>
<td>Y(4)</td>
<td>0.150420975</td>
<td>0.243659625</td>
<td>0.61734</td>
<td>0.53701011</td>
</tr>
<tr>
<td>Y(5)</td>
<td>-0.326706177</td>
<td>0.190547685</td>
<td>-1.71456</td>
<td>0.08642522</td>
</tr>
<tr>
<td>Z(1)</td>
<td>1.151502959</td>
<td>0.172813533</td>
<td>6.66327</td>
<td>0.00000000</td>
</tr>
<tr>
<td>Z(2)</td>
<td>-0.297335968</td>
<td>0.224507959</td>
<td>-1.32439</td>
<td>0.18537374</td>
</tr>
<tr>
<td>Z(3)</td>
<td>0.029945732</td>
<td>0.235624605</td>
<td>0.12709</td>
<td>0.89886489</td>
</tr>
<tr>
<td>Z(4)</td>
<td>0.134517663</td>
<td>0.231054893</td>
<td>0.58239</td>
<td>0.56043925</td>
</tr>
<tr>
<td>Z(5)</td>
<td>-0.088363661</td>
<td>0.187404894</td>
<td>-0.47142</td>
<td>0.67539791</td>
</tr>
<tr>
<td>W(1)</td>
<td>-0.262563512</td>
<td>0.197511664</td>
<td>-1.34820</td>
<td>0.17759408</td>
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<tr>
<td>W(2)</td>
<td>0.508830552</td>
<td>0.232398597</td>
<td>2.18947</td>
<td>0.02856245</td>
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<tr>
<td>W(3)</td>
<td>-0.325684940</td>
<td>0.240232223</td>
<td>-1.35463</td>
<td>0.17553490</td>
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<tr>
<td>W(4)</td>
<td>-0.091561603</td>
<td>0.242537666</td>
<td>-0.37734</td>
<td>0.7057488</td>
</tr>
<tr>
<td>W(5)</td>
<td>0.207815140</td>
<td>0.149921769</td>
<td>1.09151</td>
<td>0.27504799</td>
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</tbody>
</table>

Dependent Variable W - Estimation by Seemingly Unrelated Regressions
Quarterly Data From 1961:02 To 1984:04
Usable Observations 95 Degrees of Freedom 79
Centered R**2 0.973440 R Bar **2 0.968397
Uncentered R**2 0.997283 T x R**2 94.742
Mean of Dependent Variable -6.592059684
Std Error of Dependent Variable 2.236987385
Sum of Squared Residuals 12.49363301
Durbin-Watson Statistic 2.042989
45. \( W(2) \) & 0.648132046 & 0.249592267 & 2.59676 & 0.00941067  \\
46. \( W(3) \) & -0.461509075 & 0.258210583 & -1.78734 & 0.07388320  \\
47. \( W(4) \) & 0.014070902 & 0.260466531 & 0.05402 & 0.95691770  \\
48. \( W(5) \) & 0.193091733 & 0.204478002 & 0.94432 & 0.34500848  \\

Covariance/Correlation Matrix of Residuals  

\[
\begin{array}{ccc}
Y & Z & W \\
Y & 0.07069706105 & -0.0767572243 & 0.5672411033 \\
Z & -0.00689135929 & 0.11401707697 & 0.6157361701 \\
W & 0.05469541828 & 0.07539841081 & 0.13151192948 \\
\end{array}
\]

Chi-Squared(4) = 16.965899 with Significance Level 0.00196266

A.2.2 SAS

(1) Program

```
DATA VAR;
INFILE "EXAMPLE.DAT" ;
 INPUT Y Z W;
 YLAG1 = LAG(Y);
 YLAG2 = LAG2(Y);
 YLAG3 = LAG3(Y);
 YLAG4 = LAG4(Y);
 YLAG5 = LAG5(Y);
 ZLAG1 = LAG(Z);
 ZLAG2 = LAG2(Z);
 ZLAG3 = LAG3(Z);
 ZLAG4 = LAG4(Z);
 ZLAG5 = LAG5(Z);
 WLAG1 = LAG(W);
 WLAG2 = LAG2(W);
 WLAG3 = LAG3(W);
 WLAG4 = LAG4(W);
 WLAG5 = LAG5(W);
PROC SYSLIN SUR VARDEF=N;
TITLE "WALD TEST USING THE SUR PROCEDURE";
Y:MODEL Y = YLAG1 ZLAG1 WLAI1 YLAG2 ZLAG2 WLAI2 YLAG3 ZLAG3 WLAI3 YLAG4 ZLAG4 WLAI4 YLAG5 ZLAG5 WLAI5;
Z:MODEL Z = YLAG1 ZLAG1 WLAI1 YLAG2 ZLAG2 WLAI2 YLAG3 ZLAG3 WLAI3 YLAG4 ZLAG4 WLAI4 YLAG5 ZLAG5 WLAI5;
W:MODEL W = YLAG1 ZLAG1 WLAI1 YLAG2 ZLAG2 WLAI2 YLAG3 ZLAG3 WLAI3 YLAG4 ZLAG4 WLAI4 YLAG5 ZLAG5 WLAI5;
STEST Y,ZLAG1=0, Y,ZLAG2=0, Y,ZLAG3=0, Y,ZLAG4=0;
RUN;
```
(2) Relevant Output:

**SYSLIN Procedure**

Seemingly Unrelated Regression Estimation

Model: Y
Dependent variable: Y

**WALD TEST USING THE SUR PROCEDURE**

**SYSLIN Procedure**

Seemingly Unrelated Regression Estimation

Parameter Estimates

| Variable  | DF | Parameter Estimate | Standard Error | T for H0: Parameter=0 | Prob > |T| |
|-----------|----|--------------------|----------------|-----------------------|--------|---|
| INTERCEP  | 1  | -0.135319          | 0.099151       | -1.365                | 0.1762 |
| YLAG1     | 1  | 0.610487           | 0.163604       | 3.732                 | 0.0004 |
| ZLAG1     | 1  | -0.108172          | 0.136080       | -0.795                | 0.4290 |
| WLAG1     | 1  | 0.094815           | 0.153354       | 0.618                 | 0.5382 |
| YLAG2     | 1  | -0.119507          | 0.190519       | -0.627                | 0.5323 |
| ZLAG2     | 1  | -0.436204          | 0.176786       | -2.467                | 0.0158 |
| WLAG2     | 1  | 0.403291           | 0.182999       | 2.204                 | 0.0305 |
| YLAG3     | 1  | 0.134911           | 0.190327       | 0.709                 | 0.4805 |
| ZLAG3     | 1  | 0.090342           | 0.185539       | 0.487                 | 0.6277 |
| WLAG3     | 1  | -0.064501          | 0.189318       | -0.341                | 0.7342 |
| YLAG4     | 1  | -0.073140          | 0.191867       | -0.381                | 0.7041 |
| ZLAG4     | 1  | 0.122368           | 0.181941       | 0.673                 | 0.5032 |
| WLAG4     | 1  | 0.120483           | 0.190972       | 0.631                 | 0.5299 |
| YLAG5     | 1  | -0.108224          | 0.150044       | -0.721                | 0.4729 |
| ZLAG5     | 1  | -0.258383          | 0.147598       | -1.751                | 0.0839 |
| WLAG5     | 1  | -0.006420          | 0.149922       | -0.043                | 0.9659 |

**SYSLIN Procedure**

Seemingly Unrelated Regression Estimation

Parameter Estimates

| Variable  | DF | Parameter Estimate | Standard Error | T for H0: Parameter=0 | Prob > |T| |
|-----------|----|--------------------|----------------|-----------------------|--------|---|
| INTERCEP  | 1  | 0.089166           | 0.125916       | 0.708                 | 0.4809 |
| YLAG1     | 1  | 0.037054           | 0.207767       | 0.178                 | 0.8589 |
| ZLAG1     | 1  | 1.151503           | 0.172814       | 6.663                 | 0.0001 |
| WLAG1     | 1  | -0.262564          | 0.194751       | -1.348                | 0.1814 |
| YLAG2     | 1  | -0.332908          | 0.241949       | -1.376                | 0.1727 |
| ZLAG2     | 1  | -0.297336          | 0.224508       | -1.324                | 0.1892 |
| WLAG2     | 1  | 0.508831           | 0.232399       | 2.189                 | 0.0315 |
| YLAG3     | 1  | 0.484971           | 0.241705       | 2.005                 | 0.0484 |
| ZLAG3     | 1  | 0.029946           | 0.235625       | 0.127                 | 0.8992 |
| WLAG3     | 1  | -0.325685          | 0.240423       | -1.355                | 0.1794 |
| YLAG4     | 1  | 0.150421           | 0.243660       | 0.617                 | 0.5388 |
| ZLAG4     | 1  | 0.134518           | 0.231055       | 0.582                 | 0.5621 |
| WLAG4     | 1  | -0.091562          | 0.242524       | -0.378                | 0.7068 |
| YLAG5     | 1  | -0.326706          | 0.190548       | -1.715                | 0.0903 |
| ZLAG5     | 1  | -0.088364          | 0.187441       | -0.471                | 0.6386 |
| WLAG5     | 1  | 0.207815           | 0.190392       | 1.092                 | 0.2784 |
Model: W
Dependent variable: W

WALD TEST USING THE SUR PROCEDURE

SYSLIN Procedure
Seemingly Unrelated Regression Estimation

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | T for H0: Parameter=0 | Prob > |T| |
|----------|----|-------------------|----------------|------------------------|--------|---|
| INTERCEP| 1  | -0.094066         | 0.135232       | -0.696                 | 0.4887 |
| YLAG1    | 1  | 0.054125          | 0.223138       | 0.243                  | 0.8090 |
| ZLAG1    | 1  | 0.297994          | 0.185599       | 1.606                  | 0.1124 |
| WLAG1    | 1  | 0.553337          | 0.209160       | 2.646                  | 0.0098 |
| YLAG2    | 1  | -0.335877         | 0.259849       | -1.293                 | 0.1999 |
| ZLAG2    | 1  | -0.424655         | 0.241118       | -1.761                 | 0.0821 |
| WLAG2    | 1  | 0.648132          | 0.249592       | 2.597                  | 0.0112 |
| YLAG3    | 1  | 0.667378          | 0.259587       | 2.571                  | 0.0120 |
| ZLAG3    | 1  | 0.081160          | 0.253057       | 0.321                  | 0.7493 |
| WLAG3    | 1  | -0.461509         | 0.258211       | -1.787                 | 0.0777 |
| YLAG4    | 1  | 0.091913          | 0.261686       | 0.351                  | 0.7263 |
| ZLAG4    | 1  | 0.309089          | 0.248149       | 1.246                  | 0.2166 |
| WLAG4    | 1  | 0.014071          | 0.260467       | 0.054                  | 0.9571 |
| YLAG5    | 1  | -0.393981         | 0.204645       | -1.925                 | 0.0578 |
| ZLAG5    | 1  | -0.299372         | 0.201308       | -1.487                 | 0.1410 |
| WLAG5    | 1  | 0.193092          | 0.204478       | 0.944                  | 0.3479 |

Test:

<table>
<thead>
<tr>
<th>Numerator</th>
<th>DF</th>
<th>F Value</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.241475</td>
<td>4</td>
<td>3.5271</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

Then, \( \lambda_w = 4 \times 4.241475 = 16.965899 \)

A.2.3 SHAZAM

(1) Program:

```shazam
read(example.dat) y z w
genr ly = lag(y)
genr l2y = lag(y,2)
genr l3y = lag(y,3)
genr l4y = lag(y,4)
genr l5y = lag(y,5)
genr lz = lag(z)
genr l2z = lag(z,2)
genr l3z = lag(z,3)
genr l4z = lag(z,4)
genr l5z = lag(z,5)
genr lw = lag(w)
genr l2w = lag(w,2)
genr l3w = lag(w,3)
genr l4w = lag(w,4)
genr l5w = lag(w,5)
sample 6 100
```
system 3 / dn
ols y ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols z ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols w ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
test
test 1z:1=0
test 12z:1=0
test 13z:1=0
test 14z:1=0
end
stop

(2) Relevant Output:

EQUATION 1 OF 3 EQUATIONS
DEPENDENT VARIABLE = Y                    95 OBSERVATIONS

R-SQUARE =    .9614
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .70697E-01
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .26589
SUM OF SQUARED ERRORS-SSE=   6.7162
MEAN OF DEPENDENT VARIABLE = -4.3936
LOG OF THE LIKELIHOOD FUNCTION = -11.1823

ASYMPTOTIC
VARIABLE ESTIMATED STANDARD T-RATIO       PARTIAL STANDARDIZED ELASTICITY
NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
LY      .61049    .1636    3.732     .000     .387     .6026     .6117
LZ     -.10817    .1361    -.7949    .427     -.089    -.1416    -.0545
LW      .94815E-01  .1534    .6183     .536     .069     .1593     .1414
L2Y     -.11951    .1905    -.6273    .530     -.070    -.1172    -.1199
L2Z     -.43620    .1768    -.2467    .014     -.267    -.5729    -.2151
L2W      .40329   1.830      2.204   .028     .241    .6896     .5979
L3Y     .13491    .1903    .7088     .478     .079     .1314     .1355
L3Z    .90342E-01  .1855    .4869     .626     .055     .1187     .0436
L3W   -.64501E-01  .1893    -.3407    .733     -.038    -.1123    -.0950
L4Y    -.73140E-01  .1919    -.3812    .703     -.043    -.0709    -.0735
L4Z     .12237    .1819    .6726     .501     .075     .1599     .0577
L4W     .12048    .1910    .6309     .528     .071     .2131     .1763
L5Y   -.10822    .1500    -.7213    .471     -.081    -.1056    -.1087
L5Z   -.25838   1.476     -1.751   .080     -.193    -.3337    -.1189
L5W   -.64205E-02  .1893    -.4283E-01  .966     -.005    -.0116    -.0093
CONSTANT -.13532   .9915E-01  -1.365   .172     -.152    .0000     .0308

EQUATION 2 OF 3 EQUATIONS
DEPENDENT VARIABLE = Z                    95 OBSERVATIONS

R-SQUARE =    .9633
VARIANCE OF THE ESTIMATE-SIGMA**2 =   .11402
STANDARD ERROR OF THE ESTIMATE-SIGMA =   .33766
SUM OF SQUARED ERRORS-SSE=  10.832
MEAN OF DEPENDENT VARIABLE = -2.2553
LOG OF THE LIKELIHOOD FUNCTION = -11.1823

ASYMPTOTIC
VARIABLE ESTIMATED STANDARD T-RATIO       PARTIAL STANDARDIZED ELASTICITY
NAME    COEFFICIENT   ERROR   --------   P-VALUE CORR. COEFFICIENT  AT MEANS
LY   .37054E-01   .2078    .1783     .858     .020     .0281     .0723
LZ     1.1515    .1728     6.663     .000     .600     1.1574     1.1295
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED</th>
<th>STANDARD</th>
<th>T-RATIO</th>
<th>P-VALUE</th>
<th>COEFFICIENT CORR.</th>
<th>ELASTICITY AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
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<td>.1948</td>
<td>-1.348</td>
<td>.178</td>
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<td>L2Y</td>
<td>-.33291</td>
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<td>-1.376</td>
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<td>L2Z</td>
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<td>.2245</td>
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<td>.185</td>
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<tr>
<td>L2W</td>
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<td>-.42466</td>
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<td></td>
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</tr>
</tbody>
</table>

**EQUATION 3 OF 3 EQUATIONS**
DEPENDENT VARIABLE = W  \( R^2 = .9734 \)
95 OBSERVATIONS

**VARIANCE OF THE ESTIMATE-SIGMA**\(^2 = .13151 \)
**STANDARD ERROR OF THE ESTIMATE-SIGMA = .36265**
**SUM OF SQUARED ERRORS-SSE= 12.494**
**MEAN OF DEPENDENT VARIABLE = -6.5921**
**LOG OF THE LIKELIHOOD FUNCTION = -11.1823**

**ASYMPTOTIC**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED</th>
<th>STANDARD</th>
<th>T-RATIO</th>
<th>P-VALUE</th>
<th>COEFFICIENT CORR.</th>
<th>ELASTICITY AT MEANS</th>
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**WALD CHI-SQUARE STATISTIC = 16.965899 WITH 4 D.F. P-VALUE= .00196**