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Made Easy.

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Testing for Granger Non-Causality in Cointegrated Systems Made Easy.

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Abstract:

Considerable research has been devoted during the last five years to develop appropriate tests for Granger-Causality in integrated and cointegrated systems. Despite the existence of several tests, applied reasearch appears still to be conducted using some form of an F-test in the context of a VAR or an ECM. Reasons could be that the appropriate tests have only recently appeared in the literature, or that their implementation is relatively complex. This paper shows how to use readily available routines in RATS, SAS and SHAZAM to obtain the WALD test for Granger non-causality introduced by Toda and Yamamoto (1995).

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Testing for Granger Non-causality in Cointegrated Systems Made Easy.

1. Introduction

Testing for Granger non-causality in the context of stable VAR models involves testing whether some parameters of the model are jointly zero. In the past such testing has involved a standard F-test in a regression context.

However, recent research (see Toda and Phillips, 1993) has shown that when the variables are integrated, the F-test procedure is not valid, as the test statistic does not have a standard distribution.

Many years ago Pierce (1977) discussed the problem of ignoring the effect of autocorrelation. He concluded that “relationships will frequently tend to be ‘found’ that don’t exist ...” A similar problem occurs in this context. If inappropriate tests are used, the probability of Type I errors may well be (much) larger than the nominal size of the test. Hence, causal relationships which do not exist may be “discovered”.

Toda and Phillips’ result has given rise to the development of several alternative procedures. First, in cointegrated systems, an Error Correction model (ECM) can be transformed to its levels VAR form allowing a Wald type test (WALD) for linear restrictions to the resulting VAR model (WALD), see Lütkepohl and Reimers (1992) and Toda and Phillips (1993). Second, Mosconi and Giannini (1992) suggested a likelihood ratio test (LR) for systems that are cointegrated. Unfortunately, the virtues of simplicity and ease of application have been largely lost.

A third procedure (MWALD) (see Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996)) is theoretically very simple, as it involves estimation of a VAR model augmented in a straightforward way (MWALD). A Monte Carlo experiment which included these three alternative test procedures, presented in Zapata and Rambaldi (1997), provides evidence that the MWALD test has comparable performance in size and power to the LR and WALD tests in samples of 50 or more observations. However, as presented in the literature, the implementation of the test is not entirely straightforward and involves some programming.

Despite the existence of these tests, applied research appears still to be conducted using some form of an F-test in the context of a VAR or an Error Correction Model (see for instance: Riezman, Whitman and Summers (1996), Hewarathna and Silvapulle (1996), Shan and Sun (1996), Erenburg and Wohar (1995), Mizala and Romaguera (1995), Saunders (1995), and Thorton (1995)). The failure to reach applied economists could be due to the fact that these tests have only recently appeared in the literature or to the relative complexity of implementation. Whatever the reason, the purpose of this paper is to show how the theoretical simplicity of MWALD can be matched by easy computation using the standard facilities of common computer packages, without the need for programming. Thus, the virtue of the practical simplicity of the F-test can be re-captured when the variables are integrated or cointegrated.

The presentation is as follows: Section two sets out the notation and presents the MWALD test. Section three proves that the quadratic form required to compute the MWALD test is numerically identical to the chi-squared test obtained by estimating the model as a set of Seemingly Unrelated Regressions. Section four shows how to use the SUR routines in SHAZAM, SAS and RATS to obtain the MWALD test. Section five provides a brief summary.

2. The MWALD test for non-causality

Toda and Yamamoto (1995) proved that in integrated and cointegrated systems the Wald test for linear restrictions on the parameters of a VAR(k) has an asymptotic χ^2 distribution when a VAR(k + d_{max}) is estimated, where d_{max} is the maximum order of integration in the system.

In order to clarify the principle, let us consider the simple example of a bivariate (p=2) model, with one lag (k=1). That is,

$$x_t = A_o + A_1 x_{t-1} + e_t,$$

or more fully,

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} a_{1o} \\ a_{2o} \end{bmatrix} + \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

where $E(e_t) = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = 0$ and $E(e_t e_t') = \Sigma$.

To test that x_2 does not Granger cause x_1, we will test the parameter restriction $a_{12}^{(1)} = 0$. If now we assume that x_{1t} and x_{2t} are I(1), a standard t-test is not valid. Following Dolado and Lütkepohl (1996), we test $a_{12}^{(1)} = 0$ by constructing the usual Wald test based on least squares estimates in the augmented model:

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} a_{1o} \\ a_{2o} \end{bmatrix} + \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{11}^{(2)} & a_{12}^{(2)} \\ a_{21}^{(2)} & a_{22}^{(2)} \end{bmatrix} \begin{bmatrix} x_{1,t-2} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

We will now set down the MWALD test procedure for a general VAR(k) model.

Let x_t be a vector of p economic variables, $x_{1t}, \ldots, x_{pt}$, satisfying a VAR(k) process. Then,

$$x_t = A_o + A_1 x_{t-1} + \ldots + A_k x_{t-k} + e_t, \qquad t = k+1, \ldots, T \qquad (1)$$

where $e_{k+1}, ..., e_T$ are ~ iid $(0, \Sigma)$.

Defining $A = [A_o \; A_1 ... A_k]$ equation (1) can be written in the form:

$$x_t = A\, x1_t + e_t, \qquad t = k+1, \ldots, T \qquad (2)$$

where,

$$x1_t = [\, 1 \;\; x'_{t-1} \ldots \; x'_{t-k} \,]' \qquad (3)$$

Finally, concatenating observations horizontally by defining

$$x = [x_{k+1} \; x_{k+2} \ldots x_T]$$

$$x1 = [x1_{k+1} \;\; x1_{k+2} \; \ldots x1_T]$$

$$e = [e_{k+1} \;\; e_{k+2} \; \ldots e_T]$$

equation (2) can be written as

$$x = A\, x1 + e \qquad (4)$$

The estimate $\hat{A}$ and the variance-covariance, Σ_A, can be obtained using multivariate least squares. The estimator of A_v, where $A_v = vec(A)$, is (see Lütkepohl, 1991):

$$\hat{A}_v = ((x1\; x1')^{-1} \otimes I_p)\, vec(x)$$

and the estimated variance-covariance of $\hat{A}_v$ is given by

$$\hat{\Sigma}_A = \Gamma^{-1} \otimes \hat{\Sigma} \;, \qquad (5)$$

where,

$$\Gamma = (x1x1')/T^*$$

$$\hat{\Sigma} = x\, M\, x'/T^* \qquad (6)$$

$$M = I_{T^*} - x1'(x1\; x1')^{-1}\, x1 \qquad (7)$$

and,

$T^* = T - k$.

To compute the MWALD test we proceed as follows:

(a) Estimate a VAR($k+d_{max}$) process by multivariate least squares[2], where d_{max} is the maximum degree of integration in the system, to obtain $\hat{A}_v$ **0,** the least squares estimate of A_v.

(b) Estimate Σ_A 0using (5).

(c) Let R be a $J \times (pk+1)$ matrix which selects the appropriate 'non-causality' parameters.

Then, under the null of non-causality

$$\lambda_W = T(R\hat{A}_v)' (R\hat{\Sigma}_A R')^{-1} (R\hat{A}_v)\, 0$$

(8)

has an asymptotic $\chi^2_{(J)}$ distribution.

In the bivariate example presented above, to test that x_2 does not Granger cause x_1, we must test:

$$H_o:\; a_{12}^{(1)} = 0$$

or

$$H_o:\; R\, A_v = 0,$$

where R is given by

$$R = [0\;\; 0\;\; 0\;\; 0\;\; 1\;\; 0\;\; 0\;\; 0\;\; 0\;\; 0]$$

and A_v by

$$A_v = \left[a_{1o}\quad a_{2o}\quad a_{11}^{(1)}\quad a_{21}^{(1)}\quad a_{12}^{(1)}\quad a_{22}^{(1)}\quad a_{11}^{(2)}\quad a_{21}^{(2)}\quad a_{12}^{(2)}\quad a_{22}^{(2)}\right]'$$

2 A likelihood ratio test to empirically determine the value of 'k' is detailed in Enders (1995) pages 312-315.

3. The MWALD test in the SUR framework

The test statistic λ_W, given in (8) has been obtained by organising the observations side-by-side, as presented in (4). An alternative organisation of the data is to stack the observations vertically. This is achieved by transposing (4) to obtain

$$x' = x1' A' + e' \quad (9)$$

If we vectorise (9), we have

$$vec(x') = (I_p \otimes x1') vec(A') + vec(e') \quad (10)$$

We now introduce the following notation:

$$\alpha = vec(A')$$

$$y = vec(x')$$

$$u = vec(e')$$

enabling (9) to be written as

$$y = (I_p \otimes x1') \alpha + u \quad (11)$$

where,

$$E(uu') = \Sigma \otimes I_{T^*} \quad (12)$$

The vector y consists of observations on x_1, followed by observations on x_2 and so on. The vector α is simply a reordering of the parameter vector A_v. Equations (11) and (12) are a system of Seemingly Unrelated Regressions (SUR) (See Judge et al., 1988).

By standard theory (see, for example, Lütkepohl, 1991) the variance-covariance matrix Σ_α of the least squares estimator $\hat{\alpha}$ is estimated by

$$\hat{\Sigma}_\alpha = \hat{\Sigma} \otimes [(x1x1') / T^*]^{-1} \quad (13)$$

where $\hat{\Sigma}$ is defined in (6) and (7).

Suppose now that the matrix S selects and reorders elements of $\hat{\alpha}$ so that

$$S\hat{\alpha} = R\hat{A}_v,$$

then, it follows that

$$S\hat{\Sigma}_\alpha S' = R\ \hat{\Sigma}_A R',$$

and therefore

$$(S\hat{\alpha})'[S\hat{V}(\hat{\alpha})S']^{-1}(S\hat{\alpha}) = \lambda_W$$

where λ_W is defined in (8).

It is seen from (11) that the regressor variables in each equation of the SUR system are identical (namely, x1′) and so the least squares estimate $\hat{\alpha}$ is also the SUR estimate (see Judge et al., 1988). The form of $\hat{\Sigma}_\alpha$ given by (13) is also easily verified to be the estimated variance-covariance matrix of the SUR estimator. Thus, λ_W is just the Wald statistic for testing the restriction $S\alpha = 0$ in the SUR system (11). This test can be routinely computed by several of the available commercial econometric packages (see Section 4.).

Thus, the test for Granger non-causality becomes computationally very simple. Each variable is regressed on every variable lagged from one to $k+d_{max}$ lags in a SUR system, and the restriction $S\alpha = 0$ is tested. It should be noted that if the computer package produces an F-statistic, then $\lambda_W = JF$.

4. Using RATS, SAS, and SHAZAM to compute the MWALD

To demonstrate the computation of the MWALD test we use a system of three (p =3) variables y, z and w. The data set contains 100 observations on three I(1) variables. Pre-testing indicates that k = 4.

Let us test that z does not Granger-cause y. Because d_{max} =1, we must estimate a VAR(5) and test that z_{t-1}, z_{t-2}, z_{t-3} and z_{t-4} do not appear in the y_t equation. The system to be estimated is:

$$\begin{bmatrix} y_t \\ z_t \\ w_t \end{bmatrix} = A_o + A_1 \begin{bmatrix} y_{t-1} \\ z_{t-1} \\ w_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} y_{t-2} \\ z_{t-2} \\ w_{t-2} \end{bmatrix} + A_3 \begin{bmatrix} y_{t-3} \\ z_{t-3} \\ w_{t-3} \end{bmatrix} + A_4 \begin{bmatrix} y_{t-4} \\ z_{t-4} \\ w_{t-4} \end{bmatrix} + A_5 \begin{bmatrix} y_{t-5} \\ z_{t-5} \\ w_{t-5} \end{bmatrix} + \begin{bmatrix} e_y \\ e_z \\ e_w \end{bmatrix}$$

and the null hypothesis is

$$H_o:\ a_{12}^{(1)} = a_{12}^{(2)} = a_{12}^{(3)} = a_{12}^{(4)} = 0 \qquad (14)$$

where $a_{12}^{(i)}$ are the coefficients of z_{t-i}, i =1, . . ,4, in the first equation of the system.

If we were to compute the MWALD test as explained in Section 2., we would have to program the corresponding matrices. To illustrate, we have programmed the steps in GAUSS, a well known matrix oriented software, so that the results can be compared to the SUR output from RATS, SAS and SHAZAM. The GAUSS program and output is included in Appendix A.1 for the reader's reference. The value of λ_w is in bold.

Below we present the codes for testing the hypothesis (14) using RATS, SAS and SHAZAM. Complete output is presented in Appendix A.2 with the computed values of λ_W (F in SAS case) in bold.

4.1 RATS

```
EQUATION YEQ Y
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

EQUATION ZEQ Z
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

EQUATION WEQ W
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

SUR 3
# YEQ
# ZEQ
# WEQ
TEST(PRINT)
# 7 8 9 10
# 0 0 0 0
```

4.2. SAS

Assuming the lagged values of y, z and w have been created previously (see Appendix A.2.2 for details) the code is:

```
PROC SYSLIN SUR VARDEF=N;
TITLE "WALD TEST  USING THE SUR PROCEDURE";
Y:MODEL Y = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
            YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
Z:MODEL Z = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
            YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
W:MODEL W = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
            YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
STEST Y.ZLAG1=0, Y.ZLAG2=0, Y.ZLAG3=0, Y.ZLAG4=0;
RUN;
```

Note that in SAS the value of $\lambda_w = J \times$ 'numerator', that is the value labelled 'numerator' multiplied by the number of restrictions (J).

4.3 SHAZAM

Assuming the lagged values of y, z and w have been created previously (see Appendix A.2.3 for details) the code is:

```
sample 6 100
system 3 / dn
ols y ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols z ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols w ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
test
test lz:1=0
```

```
test l2z:1=0
test l3z:1=0
test l4z:1=0
end
```

5. Summary

This paper has presented a method for the practical implementation of the Toda and Yamamoto (1995) Wald test for Ganger non-causality in integrated and cointegrated systems. We have shown that the numerical value of the required Wald test can be obtained by using the Seemingly Unrelated Regressions routine readily available in econometrics packages. The codes for RATS, SAS and SHAZAM have been included for the reader's future reference.

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Appendix

A.1 *Computing MWALD using matrix manipulation*

(1) Program

```
/*  COMPUTING THE WALD TEST OF TODA AND YAMAMOTO(1995) AND
*  DOLADO AND LUTKEPOHL (1996).
*  THIS PROGRAM HAS BEEN WRITTEN IN GAUSS.
*  LINES BETWEEN "********"  AND MARKED "<<<<<" NEED TO BE CHANGED
*  ACCORDING TO DATA SET
****************************************************************/
load
dat[100,3] = example.dat;
         y = dat[.,1];
         z = dat[.,2];
         w = dat[.,3];
         p = 3;
         k = 5;
         t = rows(dat) - k;

 x = dat[k+1:rows(dat),1:p]';  /* VECTOR OF DEPENDENT VARIABLES */

ly = zeros(rows(dat),k);
lz = zeros(rows(dat),k);
lw = zeros(rows(dat),k);

 i = 1;
  do while i <= k;
     ly[.,i] = lagn(y,i);
     lz[.,i] = lagn(z,i);
     lw[.,i] = lagn(w,i);
   i = i + 1;
  endo;

      x1 = zeros(k*p+1,t);
 x1[1,.] = ones(1,t);

 i = 1;   j = 4;
 do while i <= k;
    x1[i+j:i*p+1,.] = (ly[k+1:rows(dat),i])'|
                      (lz[k+1:rows(dat),i])'|
                      (lw[k+1:rows(dat),i])';
                j = j + 2;
                i = i + 1;
 endo;

/*****************************************************************/
output file = mwald.out reset;

"++++++++++++WALD TEST IN AUGMENTED VAR ++++++++++++++++++++++";

vahat = ((inv(x1*x1')*x1) .*. eye(p))*vec(x);
```

```
g = (1/T)*(x1*x1');
i = eye(T);
o = (1/T)* x*(i-x1'*inv(x1*x1')*x1)*x';

siga = inv(g) .*. o;

r = zeros(4,48);             /* <<<<<<<<< CHANGE ACCORDINGLY */
r[1,7]  = 1;                 /* <<<<<<<<< CHANGE ACCORDINGLY */
r[2,16] = 1;                 /* <<<<<<<<< CHANGE ACCORDINGLY */
r[3,25] = 1;                 /* <<<<<<<<< CHANGE ACCORDINGLY */
r[4,34] = 1;                 /* <<<<<<<<< CHANGE ACCORDINGLY */

l = t*(r*vahat)'*inv(r*siga*r')*(r*vahat);

"                          VAR(k+1) OUTPUT                       ";

FORMAT /RD  /M1  12,6;
"Vector Ahat = " vahat;

"Value of MWALD = " l;

output off;
end;
```

(2) Output

```
++++++++++++WALD TEST IN AUGMENTED VAR ++++++++++++++++++++++
                          VAR(k+1) OUTPUT
Vector Ahat =
   -0.135319
    0.089166
   -0.094066
    0.610487
    0.037054
    0.054125
   -0.108172
    1.151503
    0.297994
    0.094815
   -0.262564
    0.553337
   -0.119507
   -0.332908
   -0.335877
   -0.436204
   -0.297336
   -0.424655
    0.403291
    0.508831
    0.648132
    0.134911
    0.484571
    0.667378
    0.090342
    0.029946
    0.081160
   -0.064501
   -0.325685
   -0.461509
   -0.073140
    0.150421
    0.091913
    0.122368
```

```
    0.134518
    0.309089
    0.120483
   -0.091562
    0.014071
   -0.108224
   -0.326706
   -0.393981
   -0.258383
   -0.088364
   -0.299372
   -0.006420
    0.207815
    0.193092
```

Value of MWALD =   16.965899

A.2 *Computing MWALD using the SUR routine*

A.2.1 RATS

(1) Program

```
CAL 1960 1 4
ALL 10 1990:4
OPEN DATA EXAMPLE.DAT
DATA(FORMAT=FREE,ORG=OBS) / Y Z W

EQUATION YEQ Y
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

EQUATION ZEQ Z
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

EQUATION WEQ W
# CONSTANT Y{1 2 3 4 5} Z{1 2 3 4 5} W{1 2 3 4 5}

SUR 3
# YEQ
# ZEQ
# WEQ
TEST(PRINT)
# 7 8 9 10
# 0 0 0 0
```

(2) Output

```
Dependent Variable Y - Estimation by Seemingly Unrelated Regressions
Quarterly Data From 1961:02 To 1984:04
Usable Observations     95      Degrees of Freedom    79
Centered R**2     0.961389      R Bar **2   0.954057
Uncentered R**2   0.996655      T x R**2      94.682
Mean of Dependent Variable      -4.393564211
Std Error of Dependent Variable  1.360319578
Standard Error of Estimate       0.291574095
Sum of Squared Residuals        6.7162207997
Durbin-Watson Statistic             2.018157
```

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	-0.135318861	0.099151193	-1.36477	0.17232444
2. Y{1}	0.610487113	0.163603508	3.73150	0.00019034
3. Y{2}	-0.119506609	0.190519196	-0.62727	0.53048353
4. Y{3}	0.134910599	0.190327421	0.70883	0.47842733
5. Y{4}	-0.073139742	0.191866565	-0.38120	0.70305405
6. Y{5}	-0.108223530	0.150044267	-0.72128	0.47073890
7. Z{1}	-0.108172314	0.136079742	-0.79492	0.42666087
8. Z{2}	-0.436204028	0.176785837	-2.46742	0.01360925
9. Z{3}	0.090341596	0.185539494	0.48691	0.62631997
10. Z{4}	0.122367785	0.181941134	0.67257	0.50122216
11. Z{5}	-0.258382550	0.147597829	-1.75059	0.08001742
12. W{1}	0.094814777	0.153354242	0.61827	0.53639544
13. W{2}	0.403291237	0.182999217	2.20379	0.02753935
14. W{3}	-0.064500931	0.189318103	-0.34070	0.73332843
15. W{4}	0.120482716	0.190972147	0.63089	0.52811143
16. W{5}	-0.006420483	0.149921769	-0.04283	0.96584059

```
Dependent Variable Z - Estimation by Seemingly Unrelated Regressions
Quarterly Data From 1961:02 To 1984:04
Usable Observations     95      Degrees of Freedom    79
Centered R**2     0.963283      R Bar **2   0.956312
Uncentered R**2   0.986081      T x R**2      93.678
Mean of Dependent Variable      -2.255310147
Std Error of Dependent Variable  1.771535787
Standard Error of Estimate       0.370282518
Sum of Squared Residuals        10.831622312
Durbin-Watson Statistic             2.109382
```

Variable	Coeff	Std Error	T-Stat	Signif
17. Constant	0.089165972	0.125916376	0.70814	0.47886054
18. Y{1}	0.037054231	0.207767150	0.17834	0.85845204
19. Y{2}	-0.332908069	0.241948543	-1.37595	0.16883843
20. Y{3}	0.484570553	0.241705000	2.00480	0.04498427
21. Y{4}	0.150420975	0.243659625	0.61734	0.53701011
22. Y{5}	-0.326706177	0.190547685	-1.71456	0.08642522
23. Z{1}	1.151502959	0.172813533	6.66327	0.00000000
24. Z{2}	-0.297335968	0.224507959	-1.32439	0.18537374
25. Z{3}	0.029945732	0.235624605	0.12709	0.89886849
26. Z{4}	0.134517663	0.231054893	0.58219	0.56043925
27. Z{5}	-0.088363661	0.187440849	-0.47142	0.63733971
28. W{1}	-0.262563512	0.194751166	-1.34820	0.17759408
29. W{2}	0.508830552	0.232398597	2.18947	0.02856245
30. W{3}	-0.325684940	0.240423223	-1.35463	0.17553490
31. W{4}	-0.091561603	0.242523766	-0.37754	0.70577485
32. W{5}	0.207815140	0.190392120	1.09151	0.27504799

```
Dependent Variable W - Estimation by Seemingly Unrelated Regressions
Quarterly Data From 1961:02 To 1984:04
Usable Observations     95      Degrees of Freedom    79
Centered R**2     0.973440      R Bar **2   0.968397
Uncentered R**2   0.997283      T x R**2      94.742
Mean of Dependent Variable      -6.592059684
Std Error of Dependent Variable  2.236987835
Standard Error of Estimate       0.397677328
Sum of Squared Residuals        12.493633301
Durbin-Watson Statistic             2.042989
```

Variable	Coeff	Std Error	T-Stat	Signif
33. Constant	-0.094066184	0.135232115	-0.69559	0.48668527
34. Y{1}	0.054124983	0.223138498	0.24256	0.80834451
35. Y{2}	-0.335877144	0.259848752	-1.29259	0.19615390
36. Y{3}	0.667378178	0.259587191	2.57092	0.01014284
37. Y{4}	0.091913042	0.261686426	0.35123	0.72541317
38. Y{5}	-0.393981198	0.204645077	-1.92519	0.05420526
39. Z{1}	0.297993701	0.185598889	1.60558	0.10836642
40. Z{2}	-0.424655472	0.241117851	-1.76119	0.07820546
41. Z{3}	0.081159600	0.253056946	0.32072	0.74842506
42. Z{4}	0.309088925	0.248149150	1.24558	0.21291966
43. Z{5}	-0.299372161	0.201308385	-1.48713	0.13697993
44. W{1}	0.553336982	0.209159545	2.64553	0.00815641

```
45. W{2}                     0.648132046  0.249592267      2.59676  0.00941067
46. W{3}                    -0.461509075  0.258210583     -1.78734  0.07388320
47. W{4}                     0.014070902  0.260466531      0.05402  0.95691770
48. W{5}                     0.193091733  0.204478002      0.94432  0.34500848

Covariance\Correlation Matrix of Residuals
          Y              Z              W
Y  0.07069706105  -0.0767572243   0.5672411033
Z -0.00689135929  0.11401707697   0.6157361701
W  0.05469541828  0.07539841081  0.13151192948
```

Chi-Squared(4)=   16.965899 with Significance Level 0.00196266

A.2.2 SAS

(1) Program

```
DATA VAR;
INFILE "EXAMPLE.DAT" ;
INPUT Y Z W;
YLAG1 = LAG(Y);
YLAG2 = LAG2(Y);
YLAG3 = LAG3(Y);
YLAG4 = LAG4(Y);
YLAG5 = LAG5(Y);
ZLAG1 = LAG(Z);
ZLAG2 = LAG2(Z);
ZLAG3 = LAG3(Z);
ZLAG4 = LAG4(Z);
ZLAG5 = LAG5(Z);
WLAG1 = LAG(W);
WLAG2 = LAG2(W);
WLAG3 = LAG3(W);
WLAG4 = LAG4(W);
WLAG5 = LAG5(W);
PROC SYSLIN SUR VARDEF=N;
TITLE "WALD TEST  USING THE SUR PROCEDURE";
Y:MODEL Y = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
            YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
Z:MODEL Z = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
            YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
W:MODEL W = YLAG1 ZLAG1 WLAG1 YLAG2 ZLAG2 WLAG2 YLAG3 ZLAG3 WLAG3
            YLAG4 ZLAG4 WLAG4 YLAG5 ZLAG5 WLAG5;
STEST Y.ZLAG1=0, Y.ZLAG2=0, Y.ZLAG3=0, Y.ZLAG4=0;
RUN;
```

(2) Relevant Output:

SYSLIN Procedure
Seemingly Unrelated Regression Estimation

Model: Y
Dependent variable: Y

WALD TEST USING THE SUR PROCEDURE

SYSLIN Procedure
Seemingly Unrelated Regression Estimation

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-0.135319	0.099151	-1.365	0.1762
YLAG1	1	0.610487	0.163604	3.732	0.0004
ZLAG1	1	-0.108172	0.136080	-0.795	0.4290
WLAG1	1	0.094815	0.153354	0.618	0.5382
YLAG2	1	-0.119507	0.190519	-0.627	0.5323
ZLAG2	1	-0.436204	0.176786	-2.467	0.0158
WLAG2	1	0.403291	0.182999	2.204	0.0305
YLAG3	1	0.134911	0.190327	0.709	0.4805
ZLAG3	1	0.090342	0.185539	0.487	0.6277
WLAG3	1	-0.064501	0.189318	-0.341	0.7342
YLAG4	1	-0.073140	0.191867	-0.381	0.7041
ZLAG4	1	0.122368	0.181941	0.673	0.5032
WLAG4	1	0.120483	0.190972	0.631	0.5299
YLAG5	1	-0.108224	0.150044	-0.721	0.4729
ZLAG5	1	-0.258383	0.147598	-1.751	0.0839
WLAG5	1	-0.006420	0.149922	-0.043	0.9659

Model: Z
Dependent variable: Z

WALD TEST USING THE SUR PROCEDURE

SYSLIN Procedure
Seemingly Unrelated Regression Estimation

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.089166	0.125916	0.708	0.4809
YLAG1	1	0.037054	0.207767	0.178	0.8589
ZLAG1	1	1.151503	0.172814	6.663	0.0001
WLAG1	1	-0.262564	0.194751	-1.348	0.1814
YLAG2	1	-0.332908	0.241949	-1.376	0.1727
ZLAG2	1	-0.297336	0.224508	-1.324	0.1892
WLAG2	1	0.508831	0.232399	2.189	0.0315
YLAG3	1	0.484571	0.241705	2.005	0.0484
ZLAG3	1	0.029946	0.235625	0.127	0.8992
WLAG3	1	-0.325685	0.240423	-1.355	0.1794
YLAG4	1	0.150421	0.243660	0.617	0.5388
ZLAG4	1	0.134518	0.231055	0.582	0.5621
WLAG4	1	-0.091562	0.242524	-0.378	0.7068
YLAG5	1	-0.326706	0.190548	-1.715	0.0903
ZLAG5	1	-0.088364	0.187441	-0.471	0.6386
WLAG5	1	0.207815	0.190392	1.092	0.2784

```
Model: W
Dependent variable: W
                       WALD TEST  USING THE SUR PROCEDURE

                                SYSLIN Procedure
                    Seemingly Unrelated Regression Estimation

                              Parameter Estimates
```

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-0.094066	0.135232	-0.696	0.4887
YLAG1	1	0.054125	0.223138	0.243	0.8090
ZLAG1	1	0.297994	0.185599	1.606	0.1124
WLAG1	1	0.553337	0.209160	2.646	0.0098
YLAG2	1	-0.335877	0.259849	-1.293	0.1999
ZLAG2	1	-0.424655	0.241118	-1.761	0.0821
WLAG2	1	0.648132	0.249592	2.597	0.0112
YLAG3	1	0.667378	0.259587	2.571	0.0120
ZLAG3	1	0.081160	0.253057	0.321	0.7493
WLAG3	1	-0.461509	0.258211	-1.787	0.0777
YLAG4	1	0.091913	0.261686	0.351	0.7263
ZLAG4	1	0.309089	0.248149	1.246	0.2166
WLAG4	1	0.014071	0.260467	0.054	0.9571
YLAG5	1	-0.393981	0.204645	-1.925	0.0578
ZLAG5	1	-0.299372	0.201308	-1.487	0.1410
WLAG5	1	0.193092	0.204478	0.944	0.3479

```
Test:
     Numerator:   4.241475   DF:      4  F Value:   3.5271
      Denominator: 1.202532   DF:   237   Prob>F:   0.0081
```

Then, $\lambda_W = 4 \times 4.241475 = 16.965899$

A.2.3 SHAZAM

(1) Program:

```
read(example.dat) y z w
genr ly = lag(y)
genr l2y = lag(y,2)
genr l3y = lag(y,3)
genr l4y = lag(y,4)
genr l5y = lag(y,5)
genr lz = lag(z)
genr l2z = lag(z,2)
genr l3z = lag(z,3)
genr l4z = lag(z,4)
genr l5z = lag(z,5)
genr lw = lag(w)
genr l2w = lag(w,2)
genr l3w = lag(w,3)
genr l4w = lag(w,4)
genr l5w = lag(w,5)

sample 6 100
```

```
system 3 / dn
ols y ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols z ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
ols w ly lz lw l2y l2z l2w l3y l3z l3w l4y l4z l4w l5y l5z l5w
test
test lz:1=0
test l2z:1=0
test l3z:1=0
test l4z:1=0
end
stop
```

(2) Relevant Output:

```
EQUATION  1 OF  3 EQUATIONS
 DEPENDENT VARIABLE = Y                    95 OBSERVATIONS

  R-SQUARE =    .9614
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   .70697E-01
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   .26589
 SUM OF SQUARED ERRORS-SSE=   6.7162
 MEAN OF DEPENDENT VARIABLE =  -4.3936
 LOG OF THE LIKELIHOOD FUNCTION = -11.1823
```

VARIABLE NAME	ESTIMATED COEFFICIENT	ASYMPTOTIC STANDARD ERROR	T-RATIO --------	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
LY	.61049	.1636	3.732	.000	.387	.6026	.6117
LZ	-.10817	.1361	-.7949	.427	-.089	-.1416	-.0545
LW	.94815E-01	.1534	.6183	.536	.069	.1593	.1414
L2Y	-.11951	.1905	-.6273	.530	-.070	-.1172	-.1199
L2Z	-.43620	.1768	-2.467	.014	-.267	-.5729	-.2151
L2W	.40329	.1830	2.204	.028	.241	.6896	.5979
L3Y	.13491	.1903	.7088	.478	.079	.1314	.1355
L3Z	.90342E-01	.1855	.4869	.626	.055	.1187	.0436
L3W	-.64501E-01	.1893	-.3407	.733	-.038	-.1123	-.0950
L4Y	-.73140E-01	.1919	-.3812	.703	-.043	-.0709	-.0735
L4Z	.12237	.1819	.6726	.501	.075	.1599	.0577
L4W	.12048	.1910	.6309	.528	.071	.2131	.1763
L5Y	-.10822	.1500	-.7213	.471	-.081	-.1056	-.1087
L5Z	-.25838	.1476	-1.751	.080	-.193	-.3337	-.1189
L5W	-.64205E-02	.1499	-.4283E-01	.966	-.005	-.0116	-.0093
CONSTANT	-.13532	.9915E-01	-1.365	.172	-.152	.0000	.0308

```
  EQUATION  2 OF  3 EQUATIONS
 DEPENDENT VARIABLE = Z                    95 OBSERVATIONS

  R-SQUARE =    .9633
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   .11402
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   .33766
 SUM OF SQUARED ERRORS-SSE=   10.832
 MEAN OF DEPENDENT VARIABLE =  -2.2553
 LOG OF THE LIKELIHOOD FUNCTION = -11.1823
```

VARIABLE NAME	ESTIMATED COEFFICIENT	ASYMPTOTIC STANDARD ERROR	T-RATIO --------	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
LY	.37054E-01	.2078	.1783	.858	.020	.0281	.0723
LZ	1.1515	.1728	6.663	.000	.600	1.1574	1.1295

LW	-.26256	.1948	-1.348	.178	-.150	-.3387	-.7630
L2Y	-.33291	.2419	-1.376	.169	-.153	-.2506	-.6506
L2Z	-.29734	.2245	-1.324	.185	-.147	-.2999	-.2856
L2W	.50883	.2324	2.189	.029	.239	.6681	1.4696
L3Y	.48457	.2417	2.005	.045	.220	.3624	.9480
L3Z	.29946E-01	.2356	.1271	.899	.014	.0302	.0281
L3W	-.32568	.2404	-1.355	.176	-.151	-.4354	-.9349
L4Y	.15042	.2437	.6173	.537	.069	.1120	.2945
L4Z	.13452	.2311	.5822	.560	.065	.1350	.1235
L4W	-.91562E-01	.2425	-.3775	.706	-.042	-.1243	-.2611
L5Y	-.32671	.1905	-1.715	.086	-.189	-.2447	-.6390
L5Z	-.88364E-01	.1874	-.4714	.637	-.053	-.0876	-.0792
L5W	.20782	.1904	1.092	.275	.122	.2875	.5874
CONSTANT	.89166E-01	.1259	.7081	.479	.079	.0000	-.0395

EQUATION 3 OF 3 EQUATIONS
DEPENDENT VARIABLE = W 95 OBSERVATIONS

R-SQUARE = .9734
VARIANCE OF THE ESTIMATE-SIGMA**2 = .13151
STANDARD ERROR OF THE ESTIMATE-SIGMA = .36265
SUM OF SQUARED ERRORS-SSE= 12.494
MEAN OF DEPENDENT VARIABLE = -6.5921
LOG OF THE LIKELIHOOD FUNCTION = -11.1823

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	ASYMPTOTIC T-RATIO --------	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
LY	.54125E-01	.2231	.2426	.808	.027	.0325	.0361
LZ	.29799	.1856	1.606	.108	.178	.2372	.1000
LW	.55334	.2092	2.646	.008	.285	.5653	.5501
L2Y	-.33588	.2598	-1.293	.196	-.144	-.2002	-.2246
L2Z	-.42466	.2411	-1.761	.078	-.194	-.3392	-.1396
L2W	.64813	.2496	2.597	.009	.280	.6740	.6404
L3Y	.66738	.2596	2.571	.010	.278	.3953	.4467
L3Z	.81160E-01	.2531	.3207	.748	.036	.0649	.0261
L3W	-.46151	.2582	-1.787	.074	-.197	-.4886	-.4533
L4Y	.91913E-01	.2617	.3512	.725	.039	.0542	.0616
L4Z	.30909	.2481	1.246	.213	.139	.2456	.0971
L4W	.14071E-01	.2605	.5402E-01	.957	.006	.0151	.0137
L5Y	-.39398	.2046	-1.925	.054	-.212	-.2337	-.2636
L5Z	-.29937	.2013	-1.487	.137	-.165	-.2351	-.0918
L5W	.19309	.2045	.9443	.345	.106	.2116	.1867
CONSTANT	-.94066E-01	.1352	-.6956	.487	-.078	.0000	.0143

```
|_test
|_test lz:1=0
|_test l2z:1=0
|_test l3z:1=0
|_test l4z:1=0
|_end
```

WALD CHI-SQUARE STATISTIC = 16.965899 WITH 4 D.F. P-VALUE= .00196